Probability and Statistics Solutions – Chapter 2

Problem Set 2.1 **Exercises** **Pages 33-37**

1) a) P(red) = $\frac{26}{52} = \frac{1}{2} = 0.5$

b) P(face) = $\frac{12}{52} = \frac{3}{13} \approx 0.23$

c) P(ace) = $\frac{4}{52} = \frac{1}{13} \approx 0.08$

d) P(three) = $\frac{4}{52} = \frac{1}{13} \approx 0.08$

e) P(club) = $\frac{13}{52} = \frac{1}{4} = 0.25$

f) P(three of clubs) = $\frac{1}{52} \approx 0.02$

g) P(black king) = $\frac{2}{52} = \frac{1}{26} \approx 0.04$

h) P(not a spade) = 1 - P(spade) = 1 - $\frac{13}{52} = \frac{39}{52} = \frac{3}{4} = 0.75$

2) There are 20 jelly beans in the bag.

a) P(purple) = 0

b) P(yellow) = $\frac{5}{20} = \frac{1}{4} = 0.25$

c) P(~red) = 1 - P(red) = $1 - \frac{6}{20} = \frac{14}{20} = \frac{7}{10} = 0.7$

3) a) P(three) = $\frac{1}{6} \approx 0.17$

b) P(seven) = 0

c) P(even) = $\frac{3}{6} = \frac{1}{2} \approx 0.5$

d) The prime numbers on a die are 2, 3, and 5. P(prime) = $\frac{3}{6} = \frac{1}{2} = 0.5$

e) P(a number $\geq$ 5) = $\frac{2}{6} = \frac{1}{3} \approx 0.33$

4) a) There are only 4 vowels (A, E, I, and O) on a Scattegories die.

P(vowel) = $\frac{4}{20} = \frac{1}{5} = 0.2$

b) P(~Vowel) = 1 - P(Vowel)= $1 - \frac{4}{20} = \frac{16}{20} = \frac{4}{5} = 0.8$

c) P(Q) = 0

d) P(Q^c) = 1 - P(Q) = 1 – 0 = 1

e) After Q we have R, S, T, U, V, W, X, Y, and Z. However U, V, X, Y, and Z are not on a Scattegories die. This leaves us with only 4 letters that land in our category.

P(alphabetically after Q) = $\frac{4}{20} = \frac{1}{5} = 0.2$

5) a) The weekend days are the 1st & 2nd, the 8th & 9th, the 15th & 16th, the 22nd & 23rd, and the 29th & 30th. There are 10 weekend days total. P(weekend) = $\frac{10}{31} \approx 0.32$

b) P(not a weekend) = 1 - P(weekend) = 1 - $\frac{10}{31} = \frac{21}{31} \approx 0.68$

c) P(October 31st) = $\frac{1}{31} \approx 0.03$

d) P(October 32nd) = 0

e) P(~October 31st) = 1 – P(October 31st) = 1 - $\frac{1}{31} = \frac{30}{31} \approx 0.97$

f) P(an odd-numbered day) = $\frac{16}{31} \approx 0.52$

6) a) A roulette wheel has no memory and does not care what color it drops on, therefore we get P(red) = $\frac{18}{38} = \frac{9}{19} \approx 0.47$

7) a) $\frac{2}{10} = 20\%$

b) We now have 47 heads out of 100 total flips or $\frac{47}{100} = 47\%$.

c) We now have a total of 47 + 450 = 497 heads out of 1000 flips or $\frac{497}{1000} = 49.7\%$.

d) This situation illustrates the Law of Large Numbers. As we flipped the coin more and more, the percentage of heads got closer to 50%, the theoretical percentage.

8) a)

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b) There are 6 ways to get doubles (1 & 1, 2 & 2, etc…) out of 36 results possible on the board. P(doubles) = $\frac{6}{36} = \frac{1}{6} \approx 0.17$

c) P(7) = $\frac{6}{36} = \frac{1}{6} \approx 0.17$

d) There are five ways to get 8, four ways to get 9, three ways to get 10, two ways to get 11, and one way to get 12. There are a total of fifteen ways to get a total of at least 8.

P(at least 8) = $\frac{15}{36} = \frac{5}{12} \approx 0.42$

9) a) The teacher has 22 choir members to pick from and must choose three. We assume that each soloist will sing a different song so the order they are picked will matter. There are ${}_{22}P_3 = 9,240$ ways the teacher can pick the soloists.

b) There are 15 girls to choose from and 7 boys. In order for all three of the soloists to be girls, the teacher will have to select 3 girls **&** 0 boys. There are

${}_{15}P_3 \cdot {}_7P_0 = 2,730 \cdot 1 = 2,730$ ways to do this. P(all girls) = $\frac{2,730}{9,240} = \frac{13}{44} \approx 0.30$

c) All groups of three soloists will have at least one boy except if the group of 3 has only girls in it. Therefore, P(at least one boy) = 1 – P(all girls). Use the result from part b).

P (at least one boy) = 1 - $\frac{2,730}{9,240} = \frac{6,510}{9,240} = \frac{31}{44} \approx 0.70$

10) a) There are four choices for each of the first 5 questions and then two choices for each of the next 5 questions. This gives $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 4^5 \cdot 2^5 = 1,024 \cdot 32 = 32,768$ possible answer keys.

b) Only one set of answers is completely correct or there is a $\frac{1}{32,768}$ chance of getting the entire test correct by randomly guessing.

11) a) There are 29 mowers to select from and we will choose 8. The order that they are picked will not matter as all 8 will simply be placed upon the trailer. There are $_{29}C_8 = 4,292,145$ ways to select 8 mowers.

b) In order for this to happen, we must select all six of the 48-inch deck mowers **and** any two of the remaining 23 mowers. There are $_6C_6 \cdot {_{23}C_2} = 1 \cdot 253 = 253$ ways this can be done. P(all the 48-inch deck mowers get selected) = $\frac{253}{4,292,145} = \frac{1}{16,965}$

c) Following the logic from part b), we have $_8C_2 \cdot {_{15}C_4} \cdot {_6C_2} = 28 \cdot 1,365 \cdot 15 = 573,300$ ways that we can select our eight mowers. The probability is $\frac{573,300}{4,292,145} = \frac{980}{7,337} \approx 0.13$.

Problem Set 2.1 Review Exercises

12) There are $_{11}C_3 = 165$ ways to select the committee.

13) The Fundamental Counting Principle tells us there are $5 \cdot 6 \cdot 8 = 240$ ways to do this.

14) From the grid, the most likely outcome to occur is 7.

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

15) The first contestant has 10 choices, the 2nd has 9, and the 3rd has only 8 choices. This gives a total of $10 \cdot 9 \cdot 8 = 720$ ways the contestants can choose their

Problem Set 2.2 Exercises Pages 43-46

1) Two events are independent if the outcome from one event has no influence on what the outcome of the other event will be.

2) In each case, the two cards that are dealt are independent of each other.

   a) P(red & red) = $\frac{26}{52}\cdot\frac{26}{52}=\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{4}=0.25$

   b) P(spade & spade) = $\frac{13}{52}\cdot\frac{13}{52}=\frac{1}{4}\cdot\frac{1}{4}=\frac{1}{16}\approx 0.06$

   c) P(jack & jack) = $\frac{4}{52}\cdot\frac{4}{52}=\frac{1}{13}\cdot\frac{1}{13}=\frac{1}{169}\approx 0.01$

   d) P(face & face) = $\frac{12}{52}\cdot\frac{12}{52}=\frac{3}{13}\cdot\frac{3}{13}=\frac{9}{169}\approx 0.05$

3) a) Independent

   b) Not Independent. Your first marble impacts what the second marble might be.

   c) Not Independent. Getting a raise impacts the chances of you buying a new car.

   d) Not Independent. Driving on ice increases the chances of losing control of a car.

   e) Independent

   f) Not Independent. Being a chain smoker increases your chances of lung cancer.

   g) Not Independent. Left-handed parents are more likely to raise left-handed kids.

4) These are independent events. P(blue & >3) = $\frac{1}{3}\cdot\frac{3}{6}=\frac{1}{3}\cdot\frac{1}{2}=\frac{1}{6}\approx 0.17$

5) Note that the first card impacts what the second card might be in all 3 parts.

   a) P(spade & spade) = $\frac{13}{52}\cdot\frac{12}{51}=\frac{156}{2,652}=\frac{1}{17}\approx 0.06$

   b) P(same suit) = $1\cdot\frac{12}{51}=\frac{12}{51}=\frac{4}{17}\approx 0.24$ Note that we can view this as the probability that the suit of the second card matches the first. Regardless of whether the first card is a heart, club, diamond, or spade, the chance that the second card matches it is always $\frac{12}{51}$.

   c) P(king & king) = $\frac{4}{52}\cdot\frac{3}{51}=\frac{12}{2,652}=\frac{1}{221}$

6) a) P(jack, jack, jack) = $\frac{4}{52}\cdot\frac{3}{51}\cdot\frac{2}{50}=\frac{24}{132,600}=\frac{1}{5,525}$

b) P(club, club, club) = $\frac{13}{52}\cdot\frac{12}{51}\cdot\frac{11}{50}=\frac{1,716}{132,600}=\frac{11}{850}\approx 0.01$

c) P(red, red, red) = $\frac{26}{52}\cdot\frac{25}{51}\cdot\frac{24}{50}=\frac{15,600}{132,600}=\frac{2}{17}\approx 0.12$

7) Note that probability of what your slip of paper says changes after each throw.

a) P(3 'Winner' slips) = $\frac{5}{8}\cdot\frac{4}{7}\cdot\frac{3}{6}=\frac{60}{336}=\frac{5}{28}\approx 0.18$

b) P(3 blank slips) = $\frac{3}{8}\cdot\frac{2}{7}\cdot\frac{1}{6}=\frac{6}{336}=\frac{1}{56}\approx 0.02$

8) a) P(female & female) = $\frac{18}{30}\cdot\frac{17}{29}=\frac{306}{870}=\frac{51}{145}\approx 0.35$

b) P(male & male) = $\frac{12}{30}\cdot\frac{11}{29}=\frac{132}{870}=\frac{22}{145}\approx 0.15$

c) The only way that we don't get exactly one male and one female is if we get either two males or two females. We will use our answers from parts a) and b) to get our solution. P(1 male & 1 female) = 1 – P(2 females) – P(2 males) =

$1-\frac{51}{145}-\frac{22}{145}=\frac{72}{145}\approx 0.50$

9) P(2 underweight Americans selected) = $0.18\cdot 0.18=0.18^2\approx 0.03$

10) P(both 40 or older) = $0.68\cdot 0.68=0.68^2\approx 0.46$

11) P(all 4 wear seat belts) = $0.82\cdot 0.82\cdot 0.82\cdot 0.82=0.82^4\approx 0.45$

12) P(all three tip) = $0.83\cdot 0.83\cdot 0.83=0.83^3\approx 0.57$

13) a) 75%

b) P(all three are US citizens) = $0.75\cdot 0.75\cdot 0.75=0.75^3\approx 0.42$

14) a) P(none have computers) = $0.3\cdot 0.3\cdot 0.3=0.3^3\approx 0.03$

b) P(all 3 have computers) = $0.7\cdot 0.7\cdot 0.7=0.7^3\approx 0.34$

15) P(all three cases involved weapon use) = $0.94\cdot 0.94\cdot 0.94=0.94^3\approx 0.83$

Problem Set 2.2 Review Exercises

16) i) Not Random

ii) Not Random – You probably have practiced and know good ways to throw the rock.

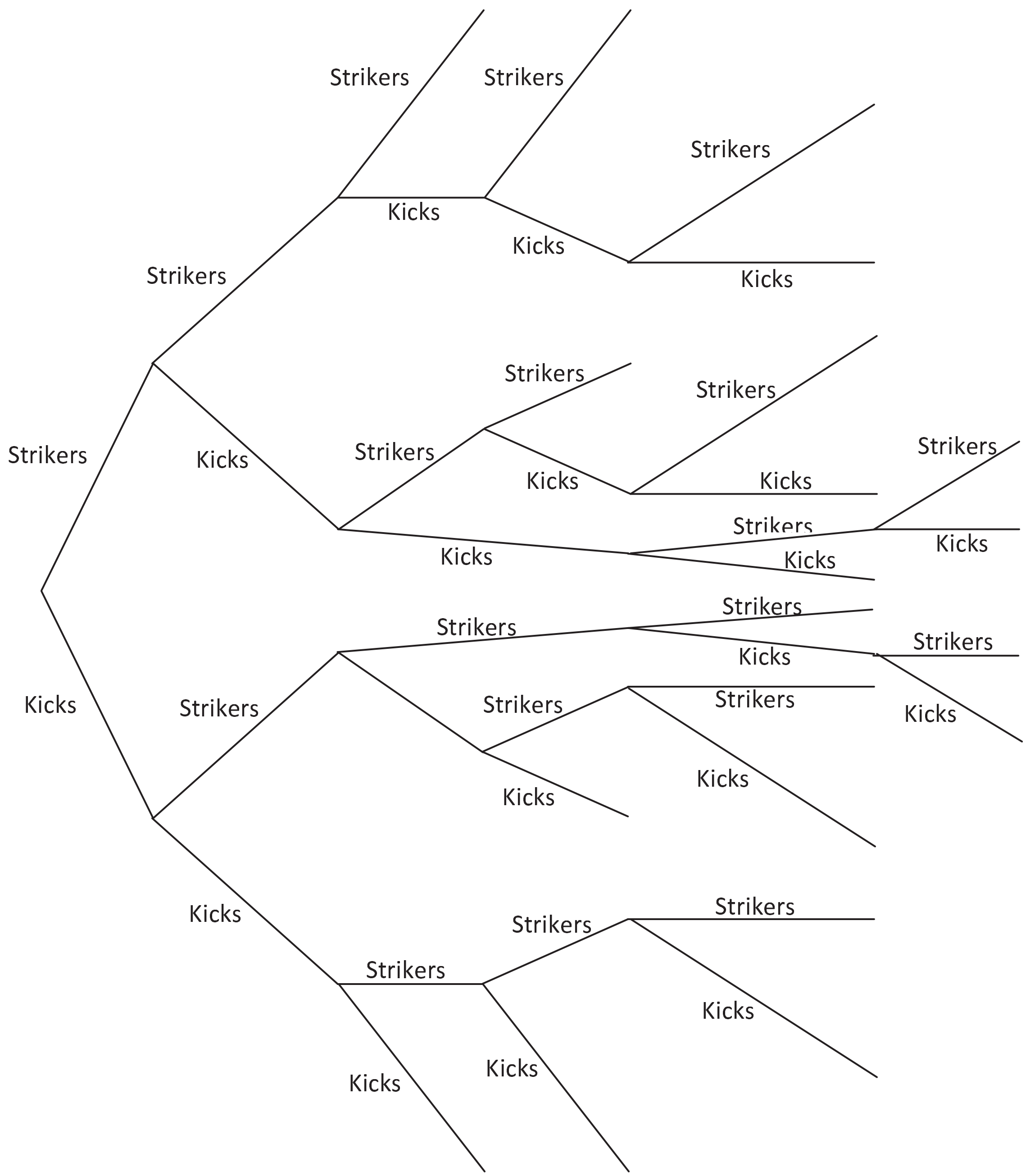
iii) Random

iv) Random

17) Since they will be giving different speeches, the order will make a difference. There are ${}_{16}P_2 = 240$ ways to choose the two speakers.

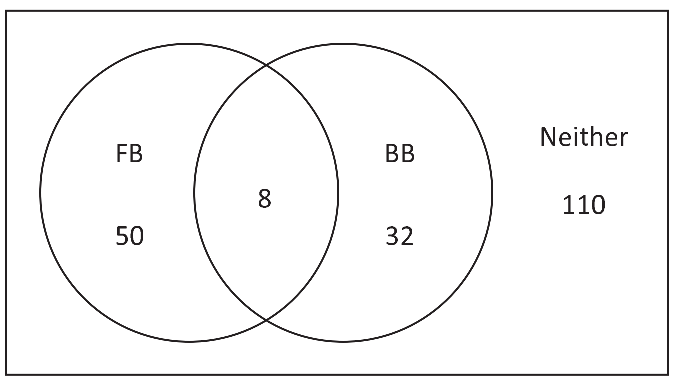
18) There are 17 choices of car for the first parent and 16 choices of car for the 2nd parent. The 1st teenager has 23 trucks to choose from and the 2nd teenager will have 22 trucks to choose from. There are $17 \cdot 16 \cdot 23 \cdot 22 = 137,632$ ways to do this.

19)



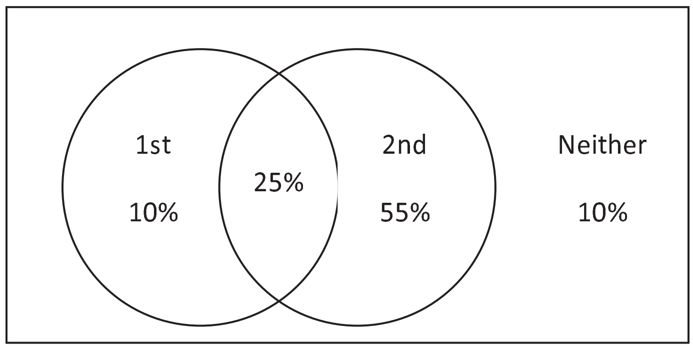
Problem Set 2.3 Exercises Pages 52-54

1) Outcomes that are mutually exclusive cannot occur at the same time from a single event.
2) Many answers possible. One example might be a die is rolled and the two mutually exclusive outcomes are getting a three and getting a four.
3) Many answers possible. One example might be a die is rolled and the two outcomes that are not mutually exclusive are getting a four and getting and even value.
4) a) Not mutually exclusive. The value of 2 lands in both outcomes.
   b) Mutually exclusive.
   c) Mutually exclusive.
   d) Not mutually exclusive. There are many students with both blond hair and blue eyes.
   e) Not mutually exclusive. There are many college students who are sophomores and also math majors.
   f) Mutually exclusive.
   g) Mutually exclusive. It is impossible for a voter to be Democrat and Republican at the same time.
5) a)



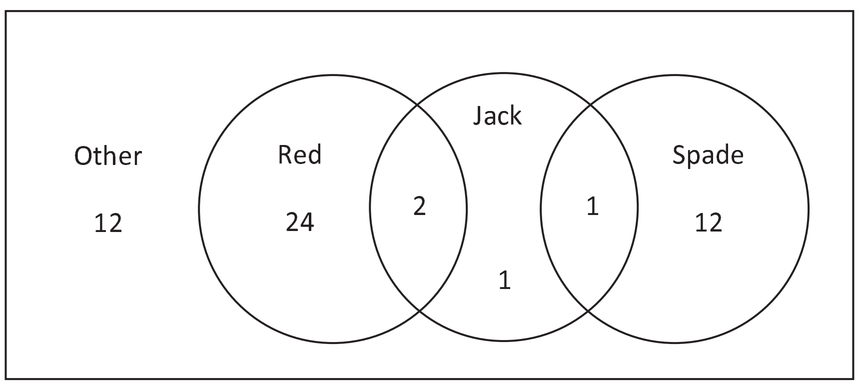
   b) 8
   c) 32
   d) 50
   e) 110

6) a) If we look at the percentages, we will note that 35% + 80% add up to 115%. This indicates that there is some overlap. That overlap must occur when the firm wins both contracts. With the 10% chance of getting neither contract, there must be a 90% chance of getting either the first or the second or both contracts. 115% - 90% = 25%, the chance of getting both jobs. The Venn Diagram is shown below.

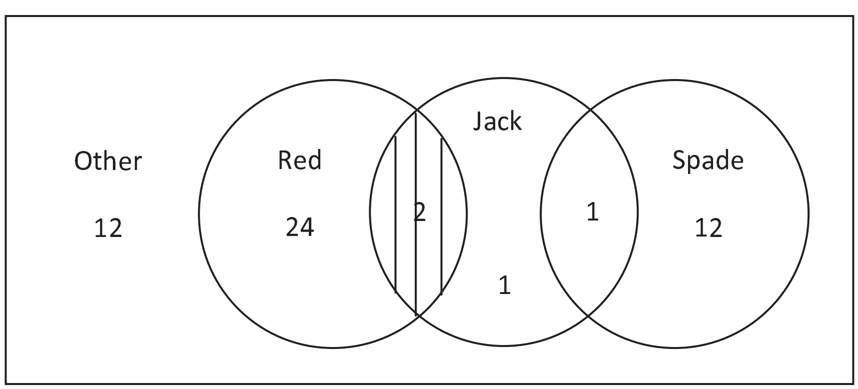


b) Use $P(1st \cup 2nd) = P(1st) + P(2nd) - P(1st \cap 2nd)$. This gives $90\% = 35\% + 80\% - P(1st \cap 2nd)$ or $-25\% = -P(1st \cap 2nd)$ or $P(1st \cap 2nd) = 25\%$.

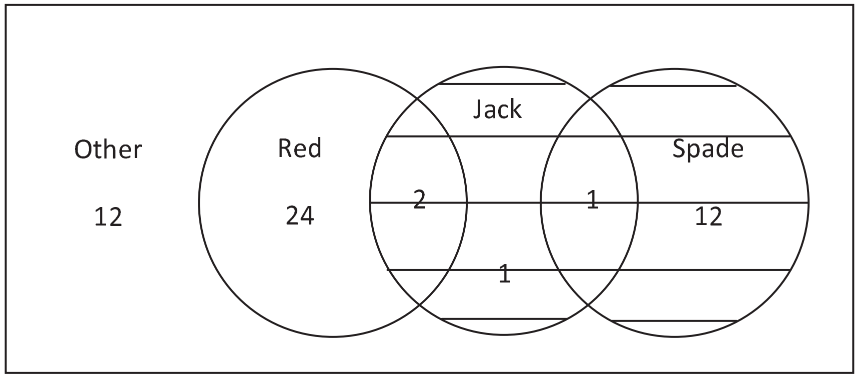
7) a)



b)



c)



8) $P(Oak \bigcup Cherry) = P(Oak) + P(Cherry) - P(Oak \bigcap Cherry)$

$P(Oak \bigcup Cherry) = 40\% + 50\% - 30\% = 60\%$

There is a 60% chance that they will use either oak, cherry, or both. This tells us that there is a 40% chance that they will use neither type of wood.

9) a) There is no overlap here so P(solid or >12) = $\frac{8}{15} + \frac{3}{15} = \frac{11}{15} \approx 0.73$

b) There are 7 even pool balls and 8 solid pool balls, however, some of these pool balls overlap so we must not count them twice. The two, four, six, and eight balls are both solid and even so there are actually only 11 pool balls total that meet our criteria.

P(even or solid) = $\frac{7}{15} + \frac{8}{15} - \frac{4}{15} = \frac{11}{15} \approx 0.73$

c) Every ball is either solid or striped so P(solid or striped) = 1

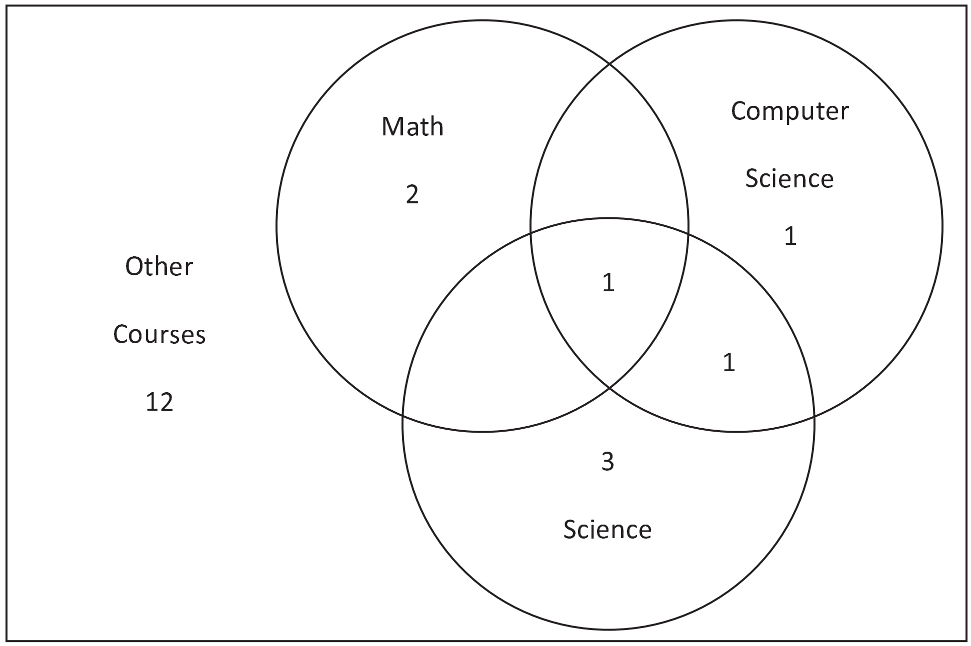
d) There are 7 striped balls and 7 even balls, however the ten, twelve, and fourteen balls are the only balls that fit into both categories. P(striped and even) = $\frac{3}{15} = \frac{1}{5} = 0.2$

10) a) P(solid & solid) = $\frac{8}{15} \cdot \frac{8}{15} = \frac{64}{225} \approx 0.28$

b) P(same ball twice) = $1 \cdot \frac{1}{15} = \frac{1}{15} \approx 0.07$ Notice that the first ball picked does not matter. There is only a one in fifteen chance that you will pick the same ball again.

c) P(solid & odd) = $\frac{8}{15} \cdot \frac{8}{15} = \frac{64}{225} \approx 0.28$

11) Based upon the diagram below, there are 12 teachers who teach other courses.



Problem Set 2.3 Review Exercises

12) The chance that both cards are face cards is $\frac{12}{52}\cdot\frac{11}{51}=\frac{132}{2652}=\frac{11}{221}\approx 0.05$.

13) $0.90\cdot 0.90\cdot 0.90\cdot 0.90=(0.90)^4\approx 0.66$

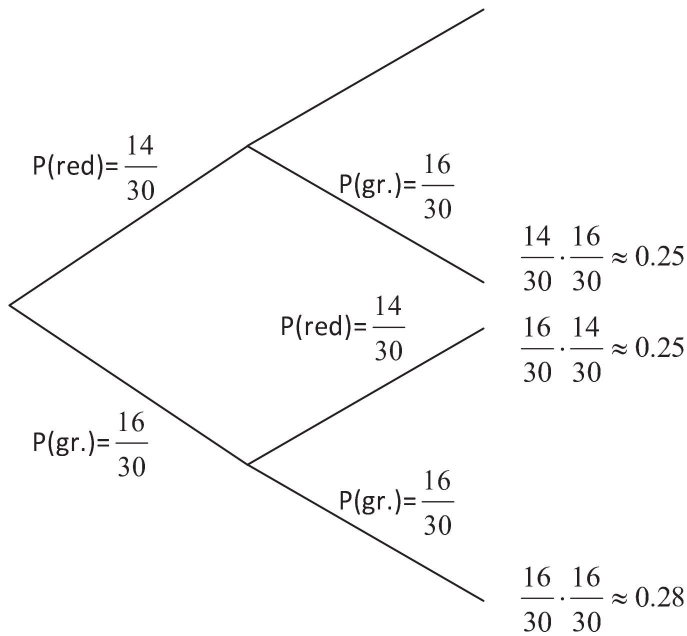
14) $P(>3\,\&\,Heart)=\frac{1}{6}\cdot\frac{13}{52}=\frac{1}{6}\cdot\frac{1}{4}=\frac{1}{24}\approx 0.04$

15) Since the order that the three crayons are selected does not matter, this will be a combination question. There are ${}_8C_3=56$ ways to select the crayons.

16) To meet the criteria of the problem, this committee must contain 1 man **&** 3 women, **or** 2 men **&** 2 women, **or** 3 men **&** 1 woman. This leads to the result below.
${}_6C_1\cdot{}_7C_3+{}_6C_2\cdot{}_7C_2+{}_6C_3\cdot{}_7C_1=6\cdot 35+15\cdot 21+20\cdot 7=210+315+140=665$

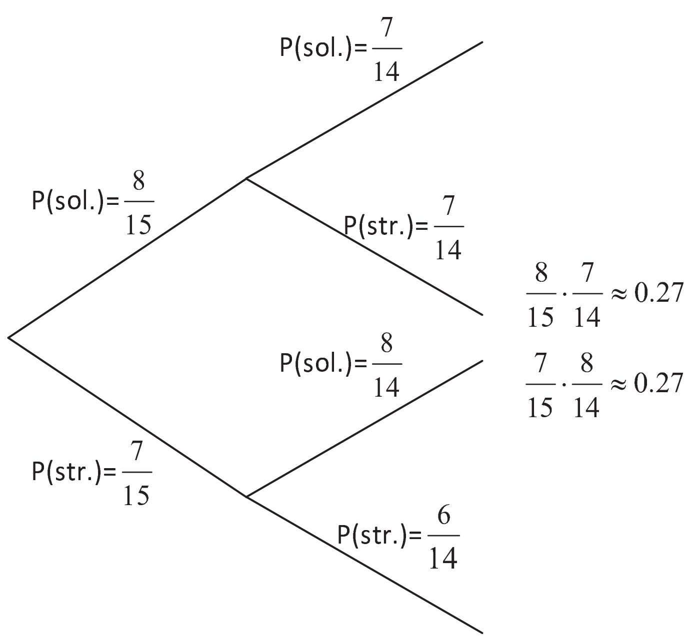
Problem Set 2.4 **Exercises** **Pages 59-60**

1)



Value	Both Red	One Red, One Green	Both Green
Probability	0.22	0.50	0.28

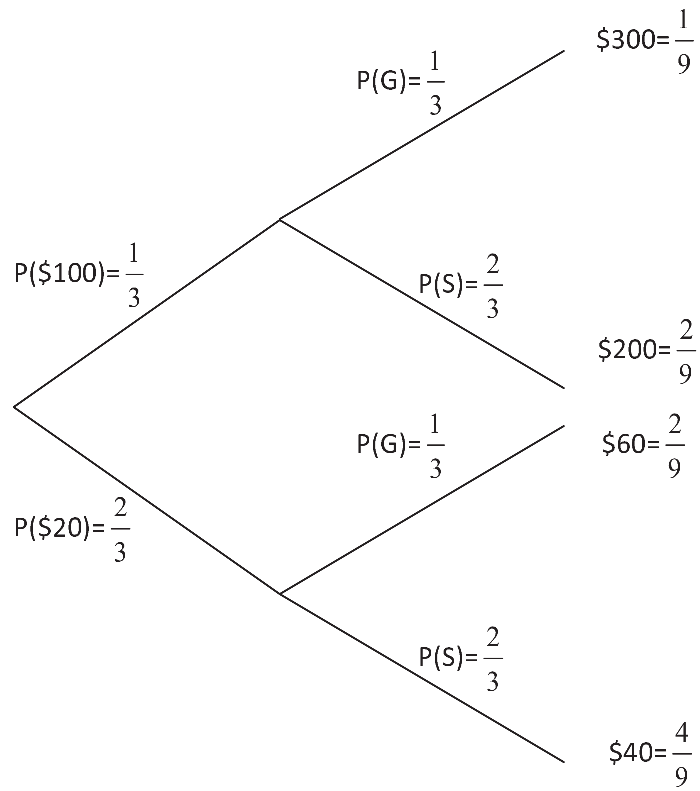
2)



We only calculate the branches we are interested in and add those together.

P(striped and solid) = $\frac{56}{210}+\frac{56}{210}=\frac{112}{210}=\frac{8}{15}\approx 0.53$

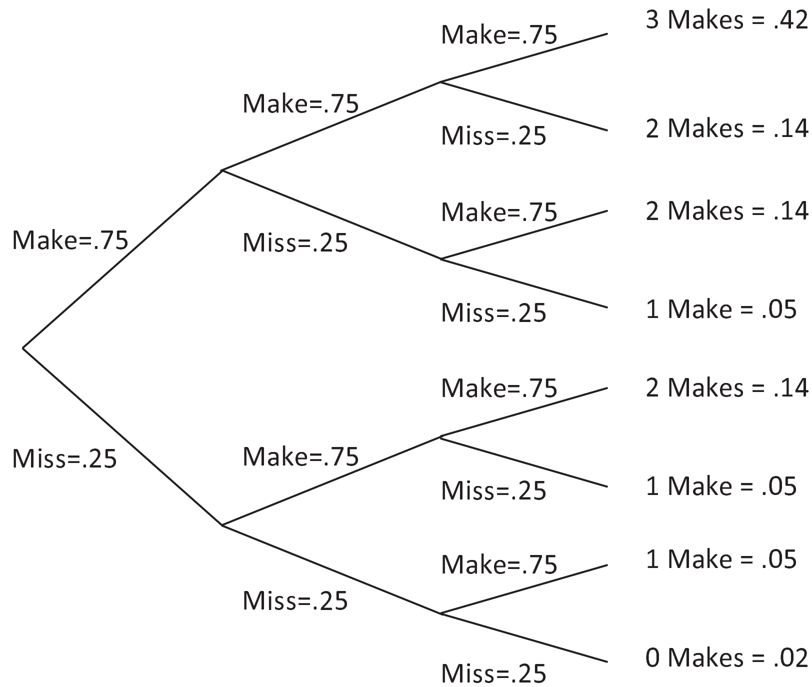
3)



Value	$40	$60	$200	$300
Probability	0.44	0.22	0.22	0.11

(Note that the probability values for the probability model above do not add up to 100% because we have used rounded values.)

4)



All values in the probability model are rounded to the nearest hundredth.

Value	0 Shots Made	1 Shot Made	2 Shots Made	3 Shots Made
Probability	.02	.14	.42	.42

5) When we flip a coin, P(heads) = $\frac{1}{2}$ and when you roll two dice, P(doubles) = $\frac{1}{6}$.

P(heads & doubles) = $\frac{1}{2}\cdot\frac{1}{6}=\frac{1}{12}\approx 0.08$

P(heads & not doubles) = $\frac{1}{2}\cdot\frac{5}{6}=\frac{5}{12}\approx 0.42$

P(tails & doubles) = $\frac{1}{2}\cdot\frac{1}{6}=\frac{1}{12}\approx 0.08$

P(tails & not doubles) = $\frac{1}{2}\cdot\frac{5}{6}=\frac{5}{12}\approx 0.42$

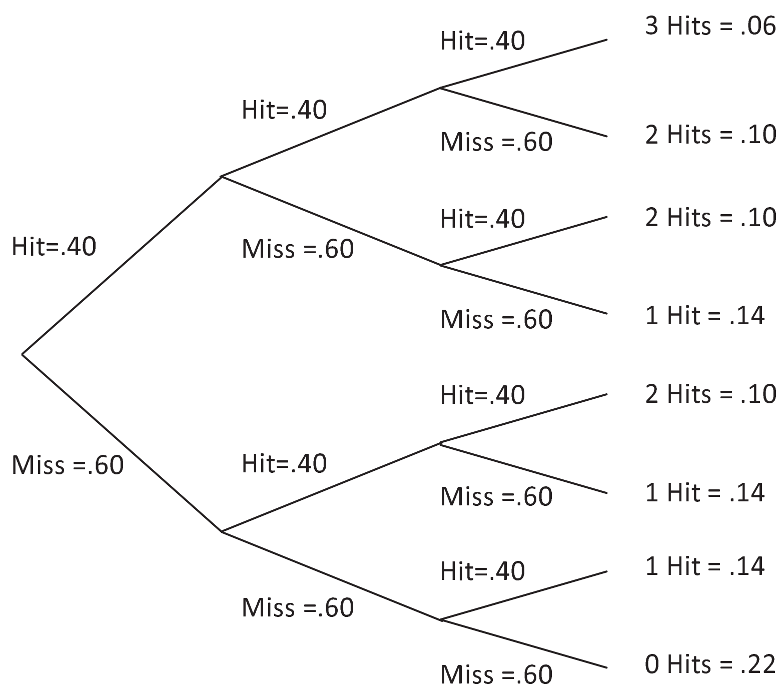
Value	Heads & Doub.	Heads & ~Doub.	Tails & Doub.	Tails & ~Doub.
Probability	0.08	0.42	0.08	0.42

6) a) There are four outcomes possible from the spinner and two from the coin. Using the Fundamental Counting Principle we get $4 \cdot 2 = 8$.

b) Note that each outcome is equally likely so the probabilities are all $\frac{1}{8}$.

Value	Red, H	Blue, H	Gr., H	Or., H	Red, T	Blue, T	Gr., T	Or., T
Prob.	0.125	0.125	0.125	0.125	0.125	0.125	0.125	0.125

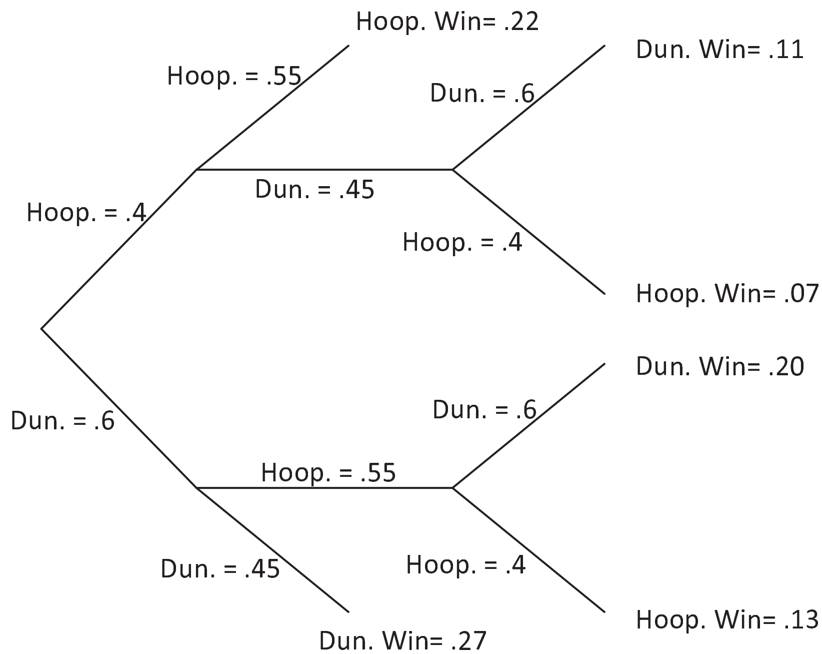
7)



All values in the probability model are rounded to the nearest hundredth.

Value	0 Hits	1 Hits	2 Hits	3 Hits
Probability	0.22	0.43	0.29	0.06

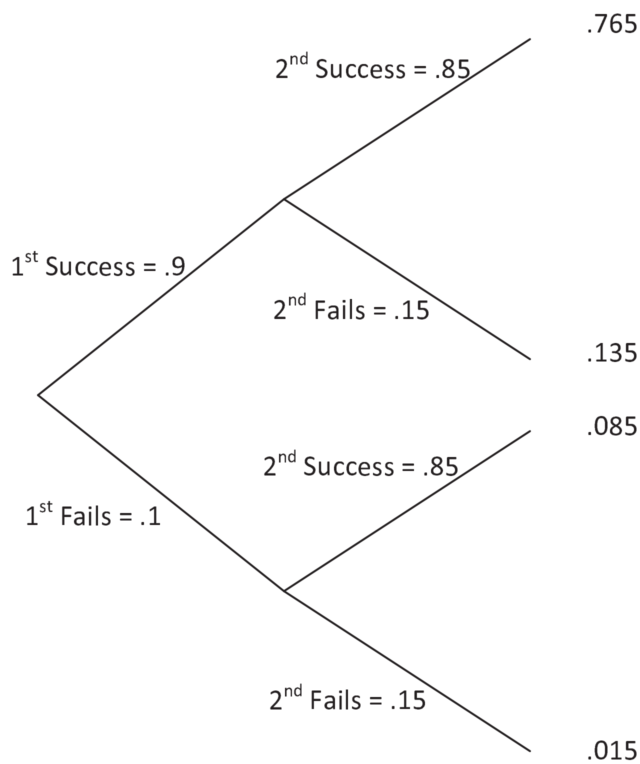
8)



All values in the probability model have been rounded to the nearest hundredth.

Value	Hoopsters Win in 2 Games	Hoopsters Win in 3 Games	Dunkers Win in 2 Games	Dunkers Win in 3 Games
Probability	0.22	0.20	0.27	0.31

9)



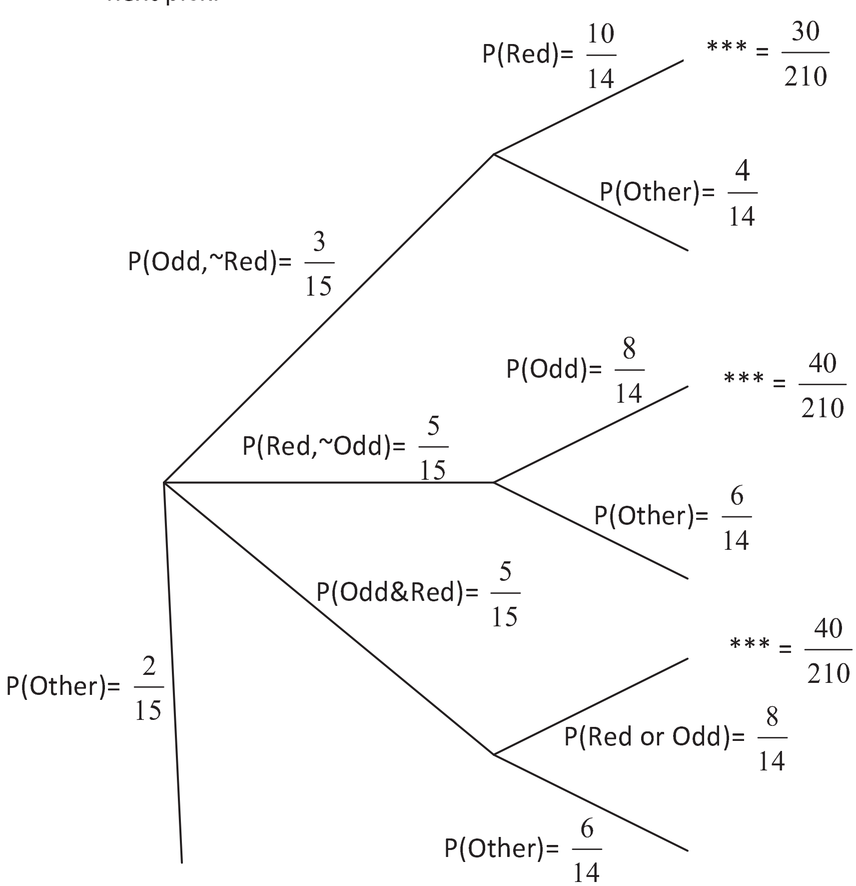
Value	Both Successes	1^{st} Success 2^{nd} Fails	1^{st} Fails 2^{nd} Success	Both Fail
Probability	0.765	0.135	0.085	0.015

10) In order to get two cubes that can be called red and odd, the first cube must be either red or odd (or both). There are three types of cubes that help us do this: a red cube that is not odd, an odd cube that is not red, and a red cube that is odd. Any other cube on the first pull results in an automatic loss.

Suppose you pull a red cube that is not odd on your first pick. You must then pull an odd cube on your next pick.

If you pull an odd cube that is not red on your first pick, you must get a red cube on your second pick.

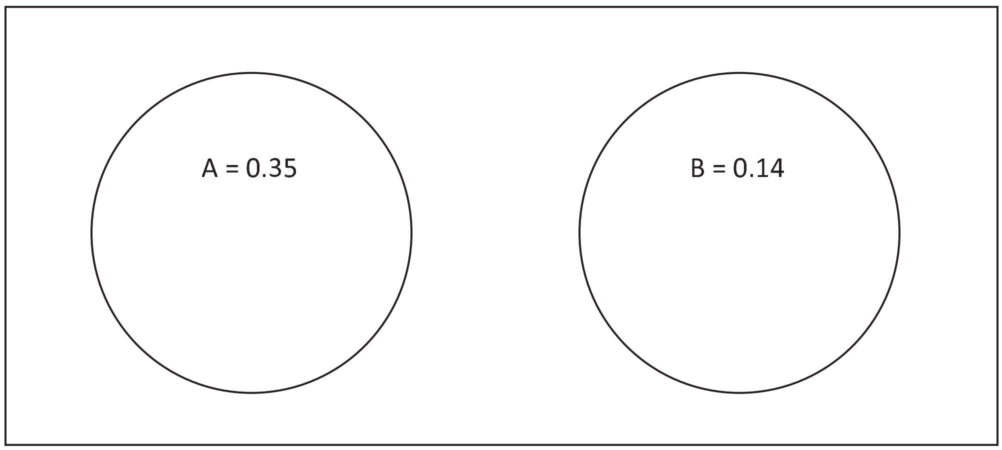
If you pull an odd and red cube on your first pick, you need only get an odd or red cube on your next pick.



The starred branches are the only ones that yield a success. Adding these gives us the probability of getting a red and an odd cube in either order. $\frac{110}{210} = \frac{11}{21} \approx 0.52$

Problem Set 2.4 Review Exercises

11) Since the two outcomes are mutually exclusive, they cannot happen at the same time. Therefore, the diagram must be like the one shown below. $P(A \cup B)$ suggests that we want anything in A, B, or both A and B. In this case $P(A \cup B)$ = 0.35 + 0.14 = 0.49.



12) There are 26 choices for the 1st letter, 25 for the 2nd, and 24 for the third so we have a total of $26 \cdot 25 \cdot 24 = 15,600$ 'words' that can be formed.

13) There are now 26 choices for each letter or $26 \cdot 26 \cdot 26 = 17,576$ 'words'.

14) Use the formula $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. We know that $P(A \cup B) = 30\%$ because if 70% of homes do not get either paper, 30% must get 1 or the other or both.

$30\% = 20\% + 15\% - P(A \cap B)$

$-5\% = -P(A \cap B)$

$5\% = P(A \cap B)$

Five percent of all homes get both newspapers.

Problem Set 2.5 Exercises Pages 66-69

1) a) $\frac{714}{1,375} \approx 0.52$

b) $\frac{661}{1,375} \approx 0.48$

c) We may only use the values from the Master's column. $\frac{128}{293} \approx 0.44$

d) We may only use the values from the Male's row. $\frac{438}{661} \approx 0.68$

2) The first card doesn't matter. We simply must insure that the 2nd, 3rd, 4th, and 5th cards match the suit of the first.

P(Flush) = $1 \cdot \frac{12}{51} \cdot \frac{11}{50} \cdot \frac{10}{49} \cdot \frac{9}{48} = \frac{11,880}{5,997,600} = \frac{33}{16,660} \approx 0.002$

3) Start by building a table that has all the totals for each row and column.

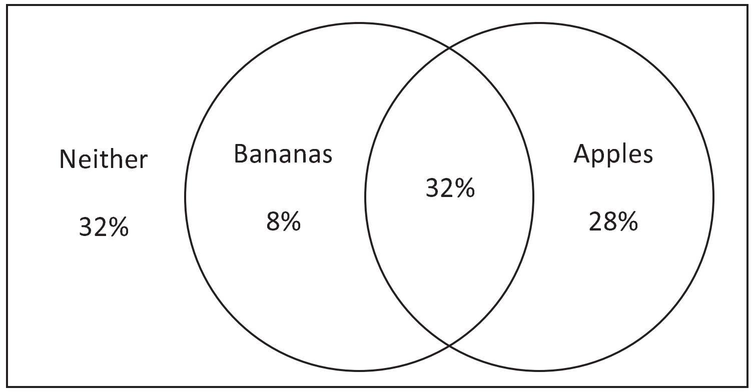
	14-17	**18-24**	**25-34**	**>34**	**Total**
Male	0.01	0.30	0.12	0.04	**0.47**
Female	0.01	0.30	0.13	0.09	**0.53**
Total	**0.02**	**0.60**	**0.25**	**0.13**	**1.00**

a) 0.53

b) We may only use values from the >34 column. $\frac{0.09}{0.13} = \frac{9}{13} \approx 0.69$

c) We must add the >34 column and the Female row. Notice that we will have to be careful not to double count the 0.09 where they intersect. 0.13 + 0.53 - 0.09 = 0.57

4) Begin by drawing a Venn Diagram for this situation



Only look at the bananas circle for this problem. P(apples|bananas) = $\frac{32}{40} = \frac{4}{5} = 0.8$

5) The next card must be a six in order to complete the straight. There are only four 6's out of the remaining 48 cards. P(6) = $\frac{4}{48} = \frac{1}{12} \approx 0.08$

6) The next card must either be a two or a seven in order to complete the straight. There are eight of these cards left in the remaining 48 cards. P(2 or 7) = $\frac{8}{48} = \frac{1}{6} \approx 0.17$

7)

	Males	**Females**	**Total**
Juniors	12	6	**18**
Seniors	4	6	**10**
Total	**16**	**12**	**28**

a) P(Junior or Female) = $\frac{24}{28} = \frac{6}{7} \approx 0.86$

b) P(Senior or Female) = $\frac{16}{28} = \frac{4}{7} \approx 0.57$

c) P(Junior or Senior) = 1

d) We may only use at the Senior row values. P(Female|Senior) = $\frac{6}{10} = \frac{3}{5} = 0.6$

8)

	Nonfiction	Fiction	Total
Children's	40	80	**120**
Adult	50	30	**80**
Total	**90**	**110**	**200**

a) $\frac{110}{200}=\frac{11}{20}=0.55$

b) $\frac{160}{200}=\frac{4}{5}=0.8$

c) $\frac{120}{200}=\frac{3}{5}=0.6$

d) We may only use values from the Nonfiction column.

P(Children's|Nonfiction) = $\frac{40}{90}=\frac{4}{9}\approx 0.44$

9) Begin this problem by building the table and give all the row and column totals.

	Mammals	Birds	Reptiles	Amphib.	Total
U.S	63	78	14	10	**165**
Foreign	251	175	64	8	**498**
Total	**314**	**253**	**78**	**18**	**663**

a) $\frac{78}{663}=\frac{2}{17}\approx 0.12$

b) There are 498 + 314 – 251 = 561 animals that are foreign or mammals.

$\frac{561}{663}=\frac{11}{13}\approx 0.85$

c) We may only use values from the U.S. row. P(Bird|U.S.) = $\frac{78}{165}=\frac{26}{55}\approx 0.47$

d) We may only use values from the foreign row. P(Bird|Foreign) = $\frac{175}{498}\approx 0.35$

10) a) There are 8 solid pools balls of which 4 have odd numbers. $\frac{4}{8}=\frac{1}{2}=0.5$

b) There are 7 seven even balls of which only the 10, 12, and 14 are striped. $\frac{3}{7}\approx 0.43$

11) Begin this problem by building the table and give all the row and column totals.

	Channel 6	Channel 8	Channel 10	Total
Quiz Show	4	2	1	**7**
Comedy	3	3	8	**14**
Drama	4	5	1	**10**
Total	**11**	**10**	**10**	**31**

a) There are 7 + 10 – 2 = 15 shows that are Quiz Shows or on Channel 8. $\frac{15}{31} \approx 0.48$

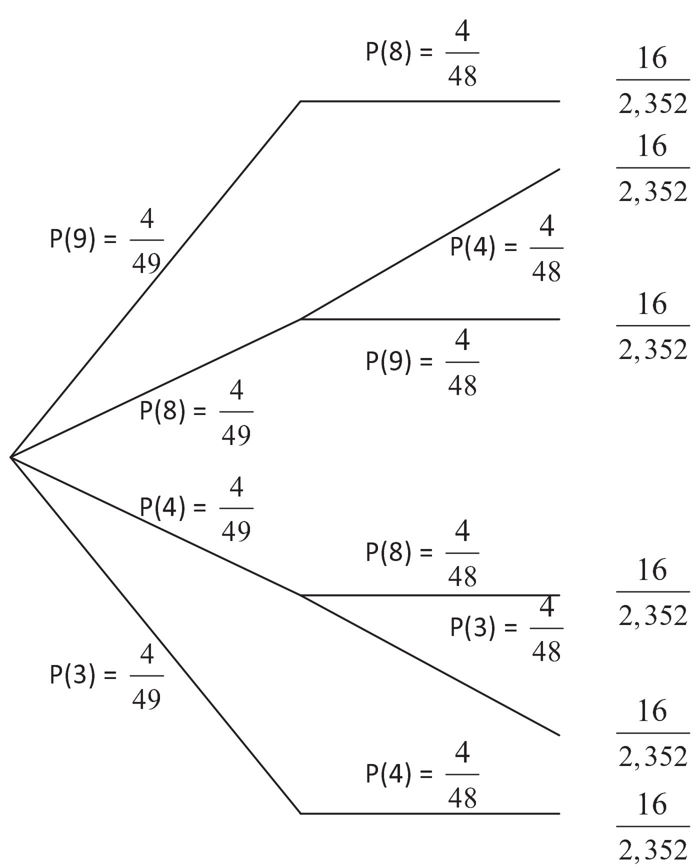
b) $\frac{24}{31} \approx 0.77$

c) We may only use values from the Channel 8 column.

P(Comedy|Channel 8) = $\frac{3}{10} = 0.3$

d) We may only use values from the drama row. P(Channel 6|Drama) = $\frac{4}{10} = \frac{2}{5} = 0.4$

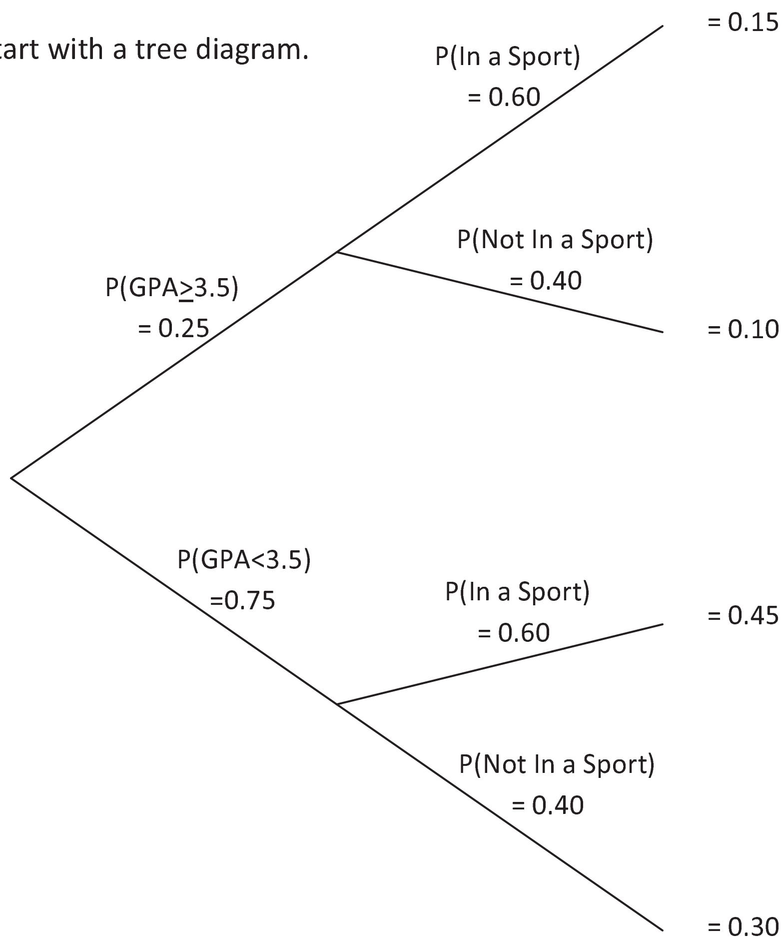
12) The tree shows all possible ways to get a straight.



Notice that the probability at the end of each branch represented in the tree has the same probability of $\frac{16}{2,352} = \frac{1}{147}$. Multiply by six to get the chance of completing the straight. $\frac{1}{147} \cdot 6 = \frac{2}{49} \approx 0.04$

Problem Set 2.4 Review Exercises

13) Start with a tree diagram.



GPA≥3.5 and in a sport	GPA≥3.5 and not in a sport	GPA<3.5 and in a sport	GPA<3.5 and not in a sport
0.15	0.10	0.45	0.30

14) There are a total of 12 face cards and 4 aces in this deck.

a) $P(Ace) = \frac{4}{16} = \frac{1}{4} = 0.25$

b) $P(Black\ Ace) = \frac{2}{16} = \frac{1}{8} = 0.125$

c) $P(Two\ Face\ Cards) = \frac{12}{16} \cdot \frac{11}{15} = \frac{132}{240} = \frac{11}{20} = 0.55$

15) The chance that the first randomly selected person was not born in the same month is $\frac{11}{12}$. This will also be true of the second person. The probability of both of these people not being born in the same month as you is $\frac{11}{12} \cdot \frac{11}{12} = \frac{121}{144} \approx 0.84$.

16) There are 4 choices for the first digit, 7 choices for the next letter, 26 choices for each of the next two letters, and 10 choices for each of the last three digits. This will give us a total of $4 \cdot 7 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 = 18{,}928{,}000$ license plates.

Chapter 2 Review Exercises

1) a) Notice that the probabilities change with each selection of another freshman or sophomore. P(3 freshmen or sophomores) = $\frac{57}{110} \cdot \frac{56}{109} \cdot \frac{55}{108} = \frac{133}{981} \approx 0.14$

b) P(3 juniors or seniors) = $\frac{53}{110} \cdot \frac{52}{109} \cdot \frac{51}{108} \approx 0.11$

c) One way to view this question is that we will be guaranteed to meet our goal if we do not have all underclassmen or all upperclassmen.
P(mixed group) = 1 – P(3 underclassmen) - P(3 upperclassmen)
P(mixed group) $\approx 1 - 0.14 - 0.11 = 0.76$

2) Begin with two charts, one for the sum and one for the product.

+	**1**	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

X	**1**	**2**	**3**	**4**	**5**	**6**
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

a) P(10 or more) = $\frac{6}{36} = \frac{1}{6} \approx 0.17$

b) P(doubles) = $\frac{6}{36} = \frac{1}{6} \approx 0.17$

c) P(total is even or less than 6) = $\frac{24}{36} = \frac{2}{3} \approx 0.67$

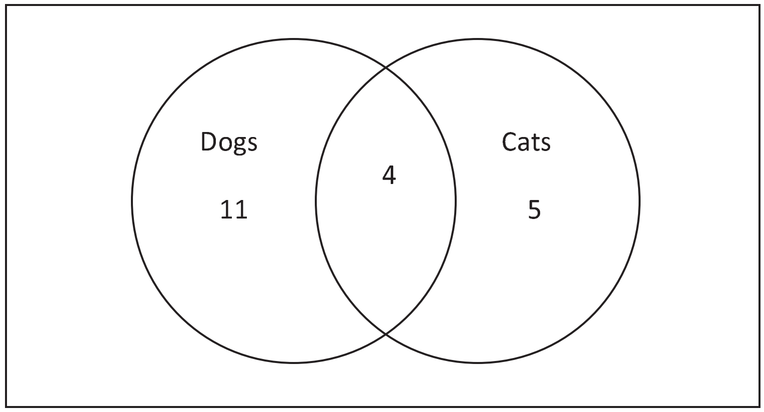
d) P(an odd product) = $\frac{9}{36} = \frac{1}{4} = 0.25$

e) P(first die is greater than second die) = $\frac{15}{36} = \frac{5}{12} \approx 0.42$

f) (Shading shown in second grid) P(a 6 or 3 is showing) = $\frac{20}{36} = \frac{5}{9} \approx 0.56$

g) (Shading shown in first grid) P(an odd total or a 2 is showing) = $\frac{23}{36} \approx 0.64$

3) a)



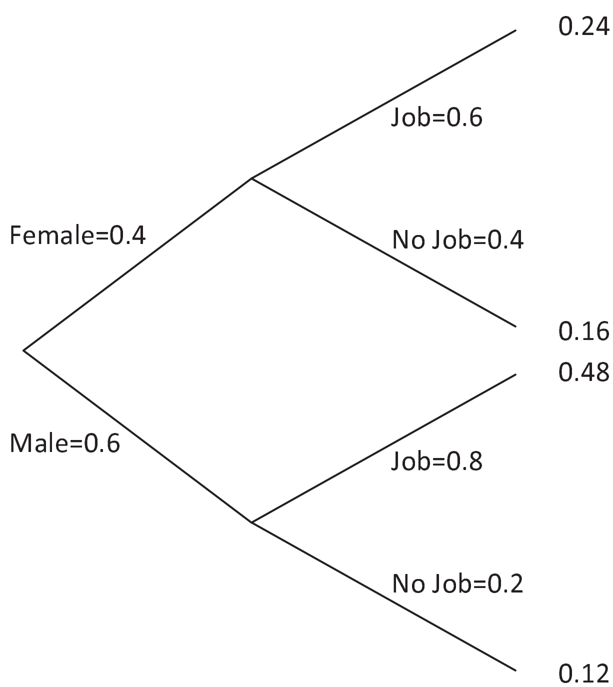
b) 20

c) $P(Dog\ Owner) = \frac{11}{20} = 0.55$

d) $P(Own\ a\ Dog \mid Own\ a\ Cat) = \frac{4}{9} \approx 0.44$

e) $P(Own\ a\ Cat \mid Own\ a\ Dog) = \frac{4}{15} \approx 0.27$

4) a)



b) 0.24 + 0.48 = 0.72

5) a) P(3 pieces) = $\frac{1}{3} \approx 0.33$

b) P(3 sugar bombs|spin a 3) = $\frac{30}{50} \cdot \frac{29}{49} \cdot \frac{28}{48} = \frac{29}{140} \approx 0.21$

c) P(3 chocolate bars|spin a 3) = $\frac{20}{50} \cdot \frac{19}{49} \cdot \frac{18}{48} = \frac{57}{980} \approx 0.06$

d) In this case the child must spin a one and then pick a chocolate bar.

P(spin a 1 and get a chocolate bar) = $\frac{1}{3} \cdot \frac{20}{50} = \frac{2}{15} \approx 0.13$

e) In this case, the child must first spin a two and then get either a sugar bomb followed by a chocolate bar or vice versa. First find the chance of getting a 2, then a sugar bomb, and finally a chocolate bar. $\frac{1}{3} \cdot \frac{30}{50} \cdot \frac{20}{49} = \frac{4}{49} \approx 0.08$ The second calculation would be spin a 2, get a chocolate bar, and then get a sugar bomb. $\frac{1}{3} \cdot \frac{20}{50} \cdot \frac{30}{49} = \frac{4}{49} \approx 0.08$

Adding these totals together gives our final result of $\frac{8}{49} \approx 0.16$.

6) First, build a contingency table for this situation including row and column totals.

	<u><</u>29	30-39	40-49	<u>></u>50	Total
Male	5	6	18	7	**36**
Female	7	7	13	4	**31**
Total	**12**	**13**	**31**	**11**	**67**

a) $P(Male) = \frac{36}{67} \approx 0.54$

b) $P(\leq 39) = \frac{25}{67} \approx 0.37$

c) $P(Male\ or\ \geq 50) = \frac{40}{67} \approx 0.60$

d) We may only use values from the Female row. $P(30-39\,|\,Female) = \frac{7}{31} \approx 0.23$

e) We may only use values from the 40-49 and <u>></u>50 columns.

$P(Female\,|\,\geq 40) = \frac{13+4}{31+11} = \frac{17}{42} \approx 0.40$

7) First, build a contingency table for this situation including row and column totals.

	GM	**Ford**	**Chrysler**	**Toyota**	**Total**
Cars	14	11	12	7	**44**
Trucks	8	9	5	6	**28**
Vans	2	3	5	3	**13**
Total	**24**	**23**	**22**	**16**	**85**

a) $\frac{23}{85} \approx 0.27$

b) $\frac{28}{85} \approx 0.33$

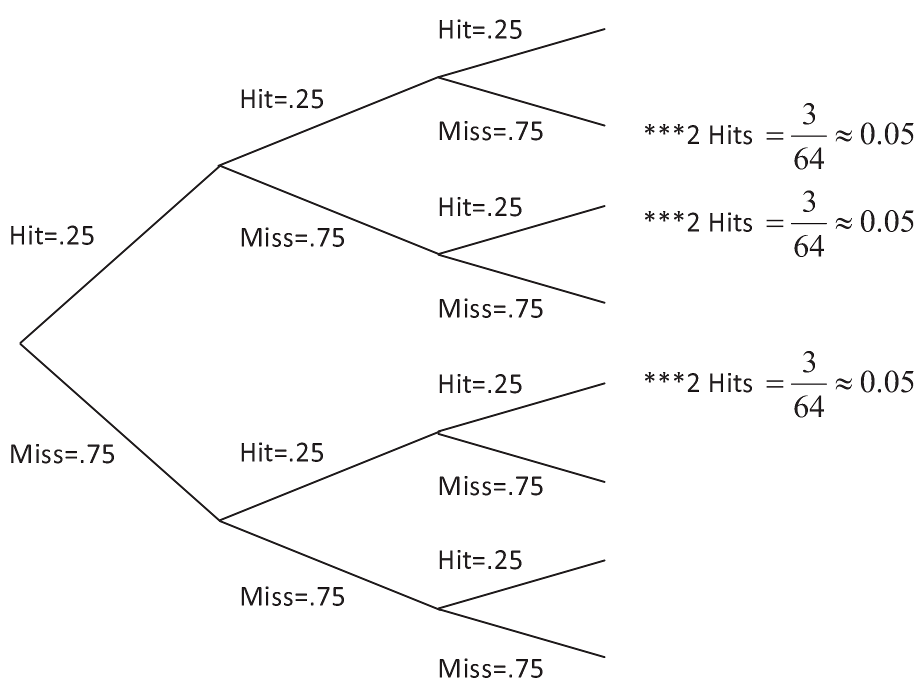
c) 13 + 16 – 3 = 26 $\frac{26}{85} \approx 0.31$

d) We may only use values from the GM column. P(Car|GM) = $\frac{14}{24} = \frac{7}{12} \approx 0.58$

e) We may only use values from the Truck row. P(Ford|Truck) = $\frac{9}{28} \approx 0.32$

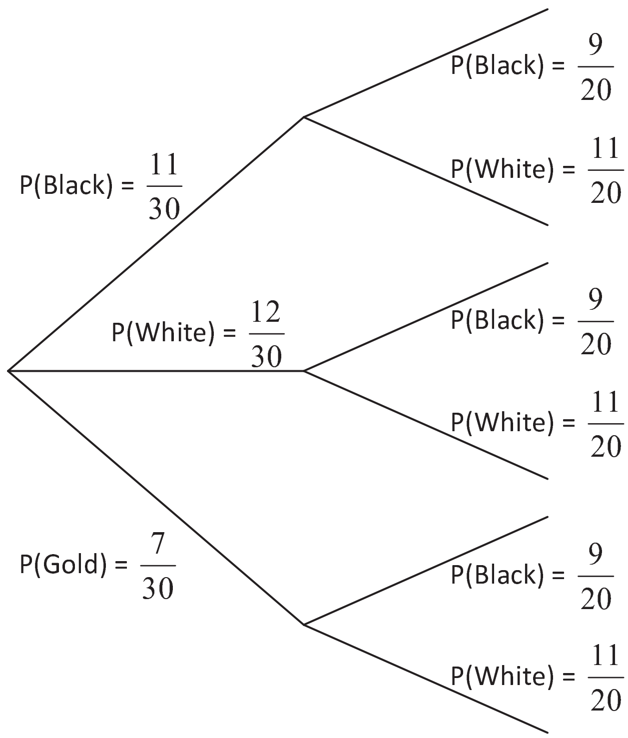
8) $P(\sim Face \& \sim Face) = \frac{40}{52} \cdot \frac{39}{51} = \frac{1560}{2652} = \frac{10}{17} \approx 0.59$

9) a)



b) In the tree diagram, we will add only the starred branches which represent the only branches that have exactly two hits on them. P(2 Hits) = $\frac{3}{64}+\frac{3}{64}+\frac{3}{64}=\frac{9}{64}\approx 0.14$

10) a)



b) Use only the top branch in the diagram. P(Both Black) = $\frac{11}{30}\cdot\frac{9}{20}=\frac{99}{600}=\frac{33}{200}\approx 0.17$

c) We now include the 'white-white' branch along with the 'black-black' branch.

P(Same Color) = $\frac{11}{30}\cdot\frac{9}{20}+\frac{12}{30}\cdot\frac{11}{20}=\frac{231}{600}=\frac{77}{200}\approx 0.39$

11) a) P(2 kings) = $\frac{2}{8}\cdot\frac{1}{7}=\frac{2}{56}=\frac{1}{28}\approx 0.04$

b) No matter what the first card is, there are 6 cards that don't match the value so we have $\frac{6}{7}\approx 0.86$.

c) No matter what the first card is, there is 1 card that will match it so we get $\frac{1}{7}\approx 0.14$.

d) No matter what the first card is, there are 3 other cards that will have the same suit so we have $\frac{3}{7}\approx 0.43$.

12) a) $P(Both\ Red) = \frac{10}{15} \cdot \frac{9}{14} = \frac{90}{210} = \frac{3}{7} \approx 0.43$

b) $P(Both\ Odd) = \frac{8}{15} \cdot \frac{7}{14} = \frac{56}{210} = \frac{4}{15} \approx 0.27$

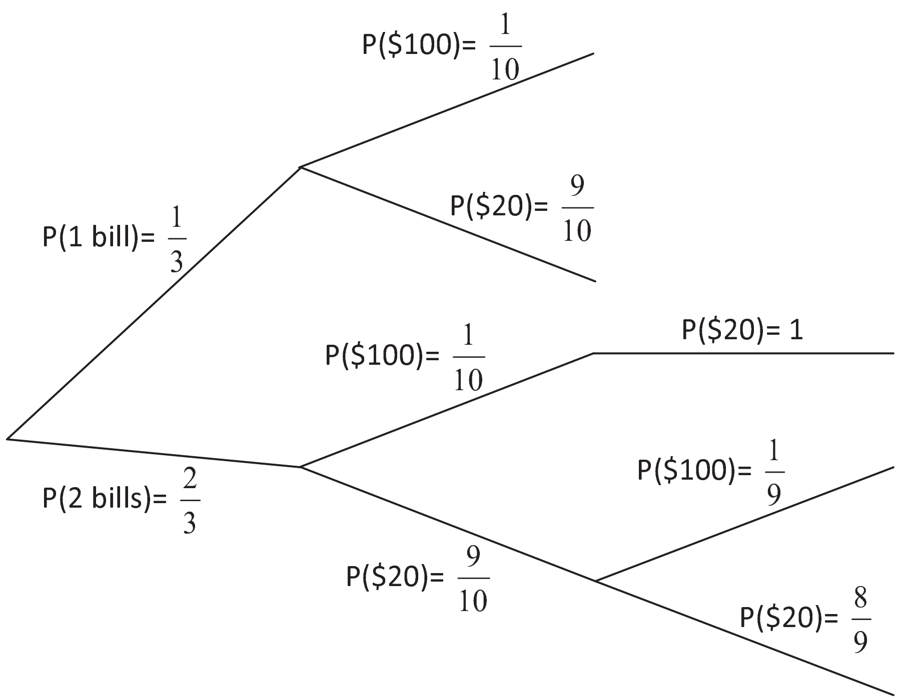
c) This means we could have green **&** green **or** red **&** red. We will have to add these results to get our solution as shown below.

$$P(Same\ Color) = \frac{5}{15} \cdot \frac{4}{14} + \frac{10}{15} \cdot \frac{9}{14} = \frac{20}{210} + \frac{90}{210} = \frac{110}{210} = \frac{11}{21} \approx 0.52$$

d) The only way for this to happen is if the first cube is a green cube or a red cube numbered 1-5. There are 10 cubes that meet this condition. Once that happens, there will only be one cube that matches the value. This gives us the calculation below.

$$P(Same\ Value) = \frac{10}{15} \cdot \frac{1}{14} = \frac{10}{210} = \frac{1}{21} \approx 0.05$$

13) a



b)

Value	$20	$40	$100	$120
Probability	0.30	0.53	0.03	0.13

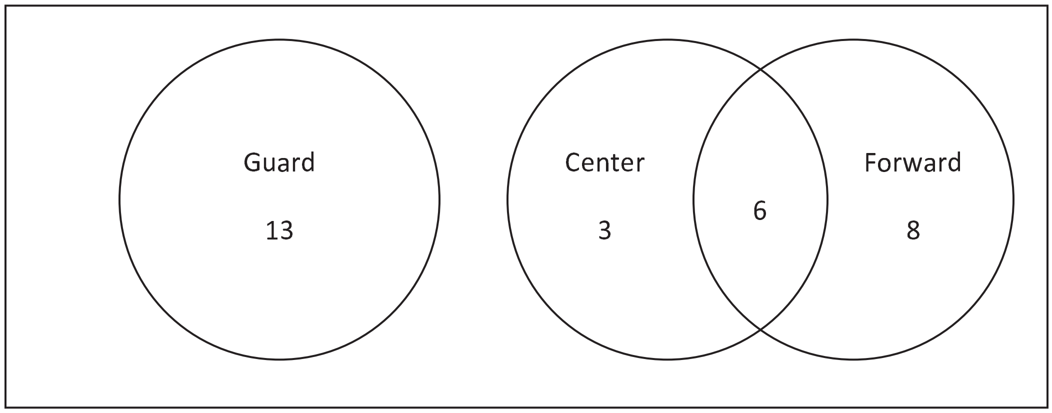
c) $P(\$120) = \frac{2}{3} \cdot \frac{1}{10} \cdot 1 + \frac{2}{3} \cdot \frac{9}{10} \cdot \frac{1}{9} = \frac{2}{30} + \frac{18}{270} = \frac{36}{270} = \frac{2}{15} \approx 0.13$

14) a) In order for the system to not detect a burglar, the burglar must get around all three parts of the system. P(no detection) = $0.30 \cdot 0.60 \cdot 0.55 \approx 0.10$

b) P(detection) = 1 – P(no detection) $\approx 1 - 0.10 = 0.90$

c) Now the burglar must beat parts 1 & 2, parts 1 & 3, or parts 2 & 3. This gives $0.30 \cdot 0.60 \cdot 0.45 + 0.30 \cdot 0.40 \cdot 0.55 + 0.70 \cdot 0.60 \cdot 0.55 \approx 0.38$

15) a)



b) P(forward) = $\frac{14}{30} \approx 0.47$

c) There were 14 players who indicated that they can play forward. Of these, 6 also indicated they can also play center. P(Center|Forward) = $\frac{6}{14} = \frac{3}{7} \approx 0.43$

16) a) $5 \cdot 6 \cdot 8 = 240$

b) She could pick three bracelets, three rings, or three necklaces.

P(3 bracelets) = $\frac{5}{19} \cdot \frac{4}{18} \cdot \frac{3}{17} = \frac{60}{5,814}$

P(3 rings) = $\frac{6}{19} \cdot \frac{5}{18} \cdot \frac{4}{17} = \frac{120}{5,814}$

P(3 necklaces) = $\frac{8}{19} \cdot \frac{7}{18} \cdot \frac{6}{17} = \frac{336}{5,814}$

P(3 like items) = $\frac{60}{5,814} + \frac{120}{5,814} + \frac{336}{5,814} = \frac{516}{5,814} = \frac{86}{969} \approx 0.09$

c) In this situation, let B=Bracelet, R=Ring, and N=Necklace. We could have BRN, BNR, RBN, RNB, NBR, NRB. We will do a couple calculations then regroup.

P(BRN) = $\frac{5}{19} \cdot \frac{6}{18} \cdot \frac{8}{17} = \frac{240}{5,814}$ P(RNB) = $\frac{6}{19} \cdot \frac{8}{18} \cdot \frac{5}{17} = \frac{240}{5,814}$

Notice that both of these (along with the other 4 scenarios) give the exact same result. There are 6 ways all together.

P(1 of each) = $6 \cdot \frac{240}{5,814} = \frac{1,440}{5,814} = \frac{80}{323} \approx 0.25$

17) a) There does not appear to be any indication that the order picked matters.
There are ${}_{36}C_3 = 7{,}140$ ways to select 3 students.

b) P(3 Juniors) = $\frac{19}{36} \cdot \frac{18}{35} \cdot \frac{17}{34} = \frac{5{,}814}{42{,}840} = \frac{19}{140} \approx 0.16$

c) Like number 16c), there will be 6 ways that this could occur and all 6 ways will be identical.

P(1 from each grade) = $6 \cdot \frac{12}{36} \cdot \frac{19}{35} \cdot \frac{5}{34} = \frac{6{,}840}{42{,}840} = \frac{19}{119} \approx 0.16$

18) a) Probably not. It is very likely that the result of the first play may have an impact on what the second play is.
b) Yes. They are mutually exclusive because it is not possible to have one play that is placed into two categories. For example, if a play is called, it can't both be a pass play and a kick play at the same time.